

# Kinematic Pile Response to Vertical P-wave Seismic Excitation

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**Abstract:** An analytical solution based on a rod-on-dynamic-Winkler-foundation model is developed for the response of piles in a soil layer subjected to vertical seismic excitation consisting of harmonic compressional waves. Closed-form solutions are derived for: (1) the motion of the pile head; (2) the peak normal strain in the pile, and (3) the group effect between neighboring piles. The solutions are expressed in terms of a dimensionless *kinematic response* factor  $I_v$ , relating pile-head motion and free-field soil surface motion, a dimensionless *strain transmissibility* factor  $I_\varepsilon$ , relating pile and soil peak normal strains, and a pile-to-pile *interaction factor*  $\alpha$  measuring group effects. It is shown that a pile foundation may significantly reduce the vertical seismic excitation transmitted to the base of a structure.

**DOI:** 10.1061/(ASCE)1090-0241(2002)128:10(860)

**CE Database keywords:** Kinematics; Excitation; Piles; Pile foundations.

## Introduction

It is well known that the seismic excitation transmitted to the base of a pile-supported structure is different (usually smaller) than the free-field motion, because of the dynamic interaction between the foundation and the surrounding soil. This interaction develops even in the absence of a superstructure and is referred to as *kinematic effect*. In the case of horizontal seismic excitation, the problem has been studied in detail (Flores-Berrones and Whitman 1982; Kaynia and Kausel 1982; Bargouthi 1984; Fan et al. 1991; Kavvadas and Gazetas 1993; Gazetas and Mylonakis 1998). On the other hand, the problem of vertical pile response has not been explored in depth (Ji and Pak 1996). It is noted that design against vertical earthquake motion is often disregarded in practice, since structures are designed to carry vertical loads. Nevertheless, evidence for a potentially detrimental role of vertical earthquake motion in the recent Northridge and Kobe earthquakes has been presented by Papazoglou and Elnashai (1996).

An issue of importance is identifying the waves present in vertical earthquake recordings. The problem is complicated and only a brief discussion is given here. Theory (Aki and Richards 1980) suggests that in the near field, within about 10 km from the epicenter, of a shallow point source in a homogeneous elastic medium, inclined SV and P waves dominate the vertical surface motions. Beyond about 20 km from source, i.e., beyond the critical angle of refraction of the SV waves, the participation of the SV component decreases substantially, while Rayleigh waves become increasingly more important.

On the other hand, engineering analyses of vertical ground motion amplification typically assume vertically propagating P waves and not inclined P-SV or Rayleigh waves (Silva 1997). This is analogous to the classical assumption in soil amplification, that horizontal earthquake motions consist exclusively of vertically propagating SH waves. As discussed by Silva (1997), the separation of wave types and the associated uncoupling of horizontal and vertical motions has been checked by comparing results of coupled nonlinear models for a number of case histories from Loma Prieta and Northridge. Results from these studies indicate that little coupling exists between vertical and horizontal motions, and that uncoupled analyses are realistic for control motions as strong as 0.5 g. Based on these findings and as a first approximation, the compressional wave model will be adopted in this work.

## Problem Definition and Model Development

The problem treated in this study is shown in Fig. 1: A single vertical pile embedded in a homogeneous soil layer resting on rigid bedrock, subjected to vertical seismic excitation. The soil is assumed to be elastic with thickness  $H$ , Young's modulus  $E_s$ , mass density  $\rho_s$ , and linear hysteretic damping  $\beta$ . The pile is a solid cylinder of length  $L$ , diameter  $d$ , and Young's modulus  $E_p$ . Perfect contact (i.e., no gap or slippage) is considered between pile and soil. The excitation consists of vertically propagating harmonic compressional waves imposed at the base of the layer. Soil-pile interaction is modeled by a bed of springs and dashpots (the springs representing soil stiffness, the dashpots energy loss due to radiation and hysteretic energy dissipation) connecting the pile to the free-field soil. Pioneered by the late Professor Novak and his coworkers in the 1970's (Novak et al. 1978), the rod-on-dynamic-Winkler-foundation model has been applied to analyze the response of piles to *lateral* kinematic loads (Flores-Berrones and Whitman 1982; Bargouthi 1984; Kavvadas and Gazetas 1993; Nikolaou et al. 2001). The analysis will be extended in this paper to the vertical mode. It should be mentioned to this end that, while dynamic Winkler formulations are well established for piles subjected to head loading, they appear less developed for piles

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Note. Discussion open until March 1, 2003. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on February 4, 2000; approved on January 7, 2002. This paper is part of the *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 128, No. 10, October 1, 2002. ©ASCE, ISSN 1090-0241/2002/10-860-867/\$8.00+\$0.50 per page.

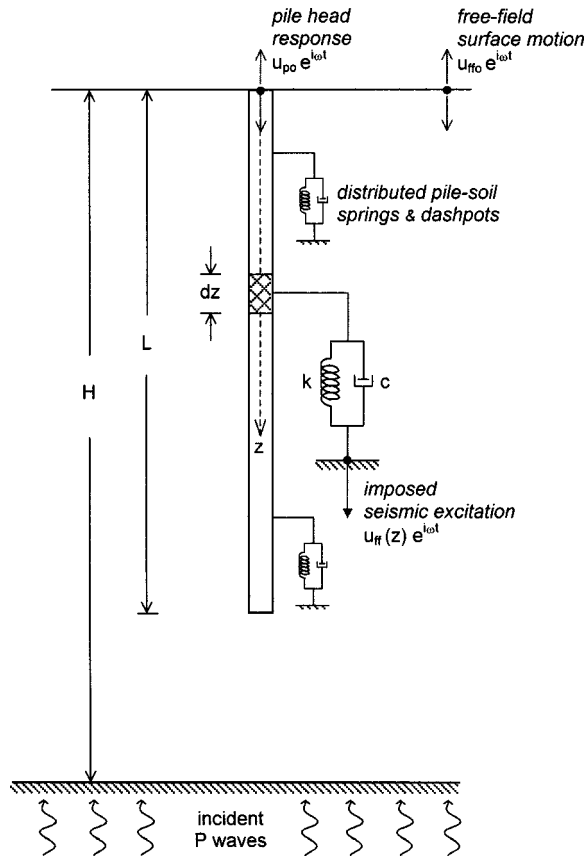


Fig. 1. System considered

subjected to seismic loads imposed directly on their shafts. To the best of the writers' knowledge, no Winkler formulations have been applied to study pile response to vertical compressional waves.

The present study deals mostly with single piles. There is evidence, documented in several analytical studies (e.g., Kaynia and Kausel 1982; Fan et al. 1991), that group effects are of secondary importance for kinematic response and, thereby, omitting them generates minor error. This, of course, is in contrast with head-loaded pile groups in which group effects may be dominant and have to be taken into account (Kaynia and Kausel 1982; Nogami and Chen 1984; Dobry and Gazetas 1988).

### Model Development

The equilibrium of vertical forces acting on the elementary pile segment of Fig. 1 is written as

$$\frac{\partial P}{\partial z} + m \frac{\partial^2 u_p}{\partial t^2} + (k + i\omega c)(u_p - u_{ff}) = 0 \quad (1)$$

where  $P = P(z)$  and  $u_p = u_p(z, t)$  denote axial force and displacement, respectively;  $u_{ff} = u_{ff}(z, t)$  = corresponding soil displacement;  $k$  and  $c$  = moduli of the distributed soil springs and dashpots;  $m$  = pile mass per unit pile length, and  $\omega$  = cyclic vibrational frequency.

Expressing the axial pile force in terms of vertical displacement,  $P = -E_p A \partial u_p / \partial z$  (compression considered positive), and restricting the analysis to harmonic vibrations,  $u_p(z, t) = u_p(z) \exp[i\omega t]$ ,  $u_{ff}(z, t) = u_{ff}(z) \exp[i\omega t]$ , Eq. (1) yields the governing differential equation

$$\frac{d^2 u_p}{dz^2} - \lambda^2 u_p = -\frac{k + i\omega c}{E_p A} u_{ff} \quad (2)$$

in which

$$\lambda = \left( \frac{k + i\omega c - m\omega^2}{E_p A} \right)^{1/2} \quad (3)$$

is a complex wave number (units = length<sup>-1</sup>) pertaining to the attenuation of pile response with depth (Novak and Aboul-Ella 1978).

The free-field soil motion,  $u_{ff}(z)$ , can be cast in the form

$$u_{ff}(z) = u_{ffo} \cos q^* z \quad (4)$$

which corresponds to a standing wave satisfying the stress-free condition at the soil surface. In the above equation,  $u_{ffo}$  = vibration amplitude at the surface, while  $q^*$  = complex wave number

$$q^* = \frac{\omega}{V_p^*} \quad (5)$$

in which  $V_p^* = V_p \sqrt{1 + 2i\beta}$  = complex propagation velocity of damped compressional waves in the soil medium.

Adopting the arguments of Novak and Aboul-Ella (1978), it is sufficient to assume that the pile toe acts as a rigid disk on the surface of a homogeneous elastic stratum of thickness equal to the distance from the pile toe to bedrock. Accordingly, the pertinent boundary condition is

$$P|_{z=L} = K_b(u_p - u_{ff})|_{z=L} \quad (6)$$

where  $K_b$  = complex dynamic impedance of the disk. The solution used by Novak and Aboul-Ella (1978) is adopted herein (Appendix I). Also, the distributed frequency-dependent springs and dashpots  $k$  and  $c$  can be taken from available solutions by Blaney et al. (1975); Novak et al. (1978); Roesset (1980), and others. This paper utilizes the finite-element-based springs and dashpots of Gazetas et al. (1992) (see Appendix I).

Enforcing the boundary condition in Eq. (6) and considering stress-free conditions at the pile head, the solution to Eq. (2) is obtained as

$$u_p(z) = u_{ffo} \left[ \frac{\Theta q^* \sin q^* L + \Omega \lambda (1 - \Theta) \cos q^* L}{\lambda (\Omega \cosh \lambda L + \sinh \lambda L)} \times \cosh \lambda z + \Theta \cos q^* z \right] \quad (7)$$

where  $\Omega$  and  $\Theta$  = dimensionless factors:

$$\Omega = \frac{K_b}{E_p A \lambda} \quad (8a)$$

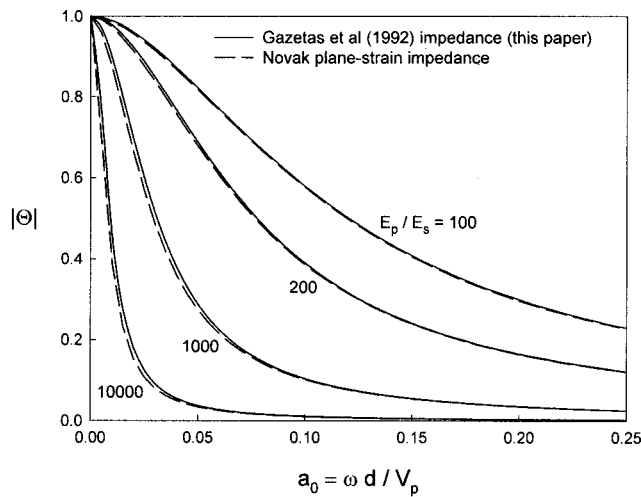
$$\Theta = \frac{k + i\omega c}{E_p A (q^{*2} + \lambda^2)} \quad (8b)$$

Of these factors,  $\Omega$  expresses a dimensionless pile toe stiffness while  $\Theta$  pertains to a particular solution of Eq. (2). As will be shown below,  $\Theta$  is related to the response of an infinitely long pile.

### Kinematic Response Factor

To develop insight into the nature of the solution, it is instructive to introduce the *kinematic response factor*

$$I_v = \frac{u_{po}}{u_{ffo}} \quad (9)$$



**Fig. 2.** Kinematic response coefficient  $\Theta$  for different pile-soil stiffness contrasts;  $\rho_s/\rho_p=0.625$ ,  $\nu_s=0.4$ , and  $\beta=0.05$ . ( $\Theta=I_v$  for infinitely-long piles.)

which relates the vibration amplitude at the pile head ( $u_{po}$ ) to that at the surface of the free-field soil ( $u_{ffo}$ ). Without soil-pile interaction,  $u_p$  would be equal to  $u_{ff}$  and  $I_v$  equal to one. In reality, however,  $u_{ff}$  and  $u_p$  are unequal in both amplitude and phase, and, thereby,  $I_v$  is generally complex. Due to space limitations, emphasis will be given to the amplitude of  $I_v$ , which suffices for most practical applications.

From Eqs. (7) and (9)  $I_v$  is obtained as

$$I_v = \frac{\Theta q^* \sin q^* L + \Omega \lambda (1 - \Theta) \cos q^* L}{\lambda (\Omega \cosh \lambda L + \sinh \lambda L)} + \Theta \quad (10)$$

in which  $\lambda$ ,  $q^*$ ,  $\Theta$ , and  $\Omega$  are given by Eqs. (3), (5), and (8), respectively. Some special cases are examined below.

For an end-bearing pile,  $\Omega \rightarrow \infty$ , Eq. (10) simplifies to

$$I_v = (1 - \Theta) \frac{\cos q^* L}{\cosh \lambda L} + \Theta \quad (11)$$

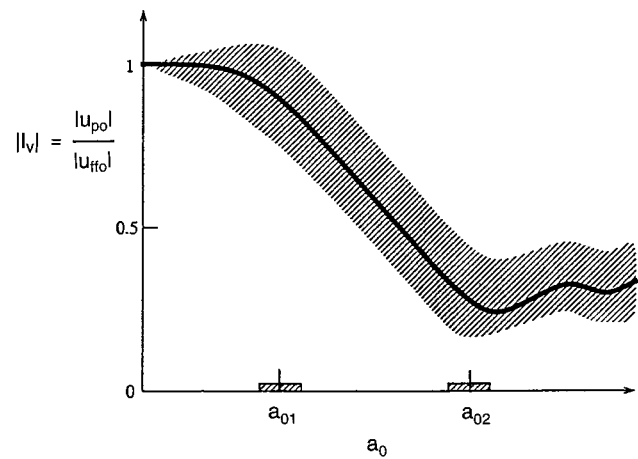
For the particular case of a pile which is completely free of reaction at the toe (termed “fully floating pile”)  $\Omega$  vanishes; thus

$$I_v = \Theta \left( \frac{q^* \sin q^* L}{\lambda \sinh \lambda L} + 1 \right) \quad (12)$$

Finally, for an infinitely long pile,  $L \rightarrow \infty$ , the hyperbolic functions in the denominators of the preceding equations become very large; all equations converge to the remarkably simple expression

$$I_v = \Theta \quad (13)$$

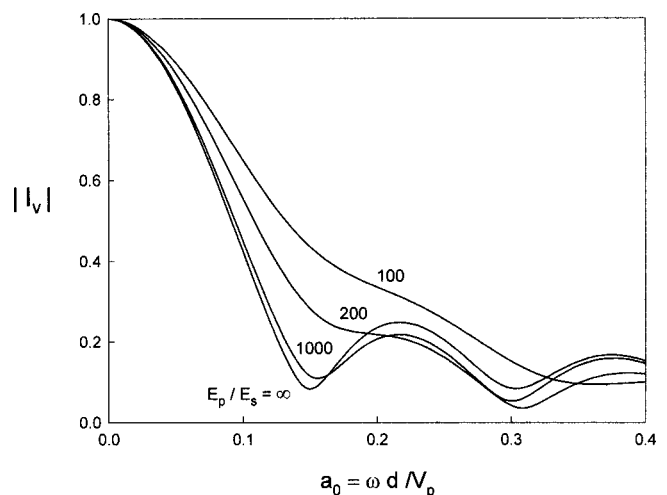
Numerical results for the dimensionless factor  $\Theta$  are presented in Fig. 2, plotted as functions of the dimensionless frequency  $a_0 = \omega d/V_p$ . It is seen that at low frequencies,  $|\Theta|$  is approximately equal to one which implies that the pile follows the free-field soil motion. With increasing frequency,  $|\Theta|$  decreases monotonically and tends to zero as  $a_0$  approaches infinity. This can be interpreted as a progressively increasing *destructive interference*, at the pile shaft of the high-frequency (short-wavelength) seismic waves exciting the pile. The trend is, understandably, stronger at large pile-soil stiffness contrasts. Corresponding predictions using the plane-strain theory of Novak are also indicated in the graph. The good agreement between the two predictions is evident confirming the insensitivity of the results to the selection of the Winkler bed.



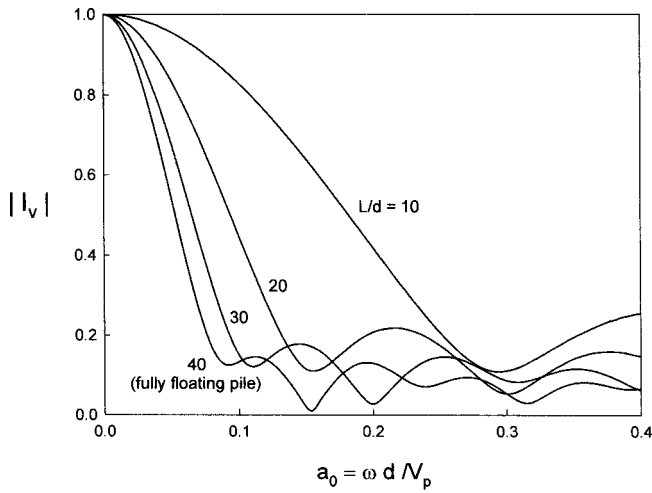
**Fig. 3.** Idealized general shape of kinematic displacement factor  $I_v$ , explaining the transition frequency factors  $a_{01}$  and  $a_{02}$  (modified after Fan et al. 1991).

With reference to piles of finite length, Fig. 3 presents the general shape of the kinematic response factor  $I_v$ . It consists of three fairly distinct regions: (1) a low-frequency region ( $0 < a_0 < a_{01}$ ) in which the “rigid-body” response of the pile to long-period seismic excitation leads to  $|I_v| \approx 1$  (see Fig. 2); (2) an intermediate frequency region ( $a_{01} < a_0 < a_{02}$ ) characterized by a rapid decline of  $|I_v|$  with increasing frequency. This behavior is a direct consequence of the progressively increasing incompatibility of the “wavy” pattern of vertical soil movement and the deformed shape of the much stiffer pile; (3) a high-frequency region ( $a_0 > a_{02}$ ) in which  $|I_v|$  fluctuates around an essentially constant value. The limiting frequencies  $a_{01}$  and  $a_{02}$  are termed (Fan et al. 1991) *first* and *second transition frequency*, respectively.

Regarding coefficient  $a_{01}$ , assuming that the transition to the intermediate frequency range occurs at the arbitrary value  $|I_v| = 0.95$ , it appears from Figs. 4 and 5 that for  $L/d \geq 20$ ,  $a_{01}$  varies between about 0.02 and 0.04. Of these values, the upper bound corresponds to “soft” compliant piles and the lower bound to stiff piles. Given that the range is relatively narrow, one may write approximately



**Fig. 4.** Amplitude of kinematic response coefficient  $I_v$  for different pile-soil stiffness contrasts;  $L/d=20$ ,  $H/L=2$ ,  $\rho_s/\rho_p=0.625$ ,  $\nu_s=0.4$ , and  $\beta=0.05$



**Fig. 5.** Amplitude of kinematic response coefficient  $I_v$  for different pile slenderness ratios;  $E_p/E_s = 1,000$ ,  $H/L = 40$ ,  $\rho_s/\rho_p = 0.625$ , and  $\nu_s = 0.4$ ,  $\beta = 0.05$

$$a_{01} \approx 0.03, \quad L/d \geq 20 \quad (14)$$

As evident from Fig. 5, coefficient  $a_{02}$  attains a much wider range of values. Accurate estimation of  $a_{02}$  is important in the design of pile-supported structures because only frequencies smaller than  $a_{02}$  will be transmitted to the superstructure; any higher frequencies contained in the free-field soil motion will be essentially “filtered out” by the pile.

To determine  $a_{02}$ , it is observed from Eq. (10) that  $|I_v|$  attains a minimum when the numerator in the first term in the right side of the equation is zero. This leads to the indicial equation

$$(a_{02})_n = \left\{ n\pi + \arctan \left[ \frac{-\Omega(1-\Theta)\lambda}{\Theta q^*} \right] \right\} \left( \frac{L}{d} \right)^{-1} \quad (15)$$

where  $n$  = positive integer ( $n = 1, 2, 3, \dots$ ). In principle, the smallest frequency,  $(a_{02})_1$ , will be the desired transition frequency  $a_{02}$ ; the higher roots would define additional minima as shown in Figs. 4 and 5. Note that, since  $\Omega$ ,  $\Theta$ ,  $\lambda$ , and  $q^*$  are functions of frequency and  $a_{02}$  is unknown, an iterative procedure is generally required to get  $a_{02}$  from Eq. (15).

An interesting special case is obtained with stiff piles. For such piles, both  $\lambda$  and  $\Omega$  are small [recall that Eqs. (3) and (8a) are proportional to the inverse of  $\sqrt{E_p}$ ]. Accordingly, the arctan term in Eq. (15) can be omitted leading to the simple expression

$$a_{02} \approx \pi \left( \frac{L}{d} \right)^{-1} \quad (16)$$

which was obtained for  $n = 1$ .

As an example, for slenderness ratios  $L/d = 10, 20, 30$ , and  $40$ , Eq. (16) yields, respectively, the values  $0.31, 0.16, 0.10$ , and  $0.08$  which are almost identical to those observed in Fig. 5. The above result also holds for fully floating piles ( $\Omega = 0$ ) regardless of pile-soil stiffness contrast.

A simpler way to obtain the result in Eq. (16) is to consider that a perfectly rigid pile tends to remain motionless when the wavelength  $\lambda$  of the imposed seismic wave is equal to two times the pile length  $L$ . From the fundamental relation  $\omega = 2\pi V/\lambda$ , substituting  $\lambda = 2L$  and multiplying both sides by  $d/V$  leads to Eq. (16).

Incidentally, it is noted that the kinematic response coefficient of a perfectly rigid pile is

$$I_v = \frac{(k + i\omega c) \frac{1}{q^*} \sin q^* L + K_b \cos q^* L}{(k + i\omega c - m\omega^2)L + K_b} \quad (17)$$

which reveals the dependence of  $I_v$  on the pile toe stiffness  $K_b$ .

In the case of soft piles (say  $E_p/E_s < 200$ ), it is observed from Fig. 4 that  $a_{02}$  occurs at higher frequencies than for a stiff pile of the same length. This can be explained by recalling that with soft piles  $|\Theta|$  attenuates slowly with frequency (see Fig. 2), so at frequency  $(a_{02})_1$ ,  $|I_v|$  is still quite high and continues to decrease with increasing  $a_0$ . For such piles, a good estimate of  $a_{02}$  can be obtained by considering the *second* harmonic  $(a_{02})_2$  in Eq. (15). Accordingly,

$$a_{02} = \left\{ 2\pi + \arctan \left[ \frac{-\Omega(1-\Theta)\lambda}{\Theta q^*} \right] \right\} \left( \frac{L}{d} \right)^{-1} \quad (18)$$

As a first approximation,  $\Omega$ ,  $\Theta$ ,  $\lambda$ , and  $q^*$  in Eq. (18) can be evaluated for  $a_0 = 2\pi(L/d)^{-1}$ , which is obtained from Eq. (18) by neglecting the arctan term. Considering a soft pile with  $L/d = 20$  and  $E_p/E_s = 100$  and  $200$ , Eq. (18) yields, respectively:  $a_{02} = 0.33$  and  $0.32$ , which are in good agreement with the observed values of  $0.35$  and  $0.31$  in Fig. 4.

Additional insight on the physical problem can be gained by comparing the transition frequency  $a_{02}$  with the fundamental natural frequency of the soil layer in compression extension,  $a_{0, \text{res}}$ . In a homogeneous layer,

$$a_{0, \text{res}} = \frac{\pi}{2} \left( \frac{H}{d} \right)^{-1} \quad (19)$$

Dividing Eqs. (16) and (19) yields the simple expression

$$\frac{a_{02}}{a_{0, \text{res}}} = 2 \frac{H}{L} \quad (20)$$

which indicates that for a stiff pile, the second transition frequency will be at least two times the fundamental natural frequency in vertical compression extension of the soil layer.

A comparison against a rigorous elastodynamic solution by Ji and Pak (1996) is depicted in Fig. 6. It refers to a single hollow pile of wall thickness  $h$  and variable slenderness  $(L/d)$ , embedded in a homogeneous halfspace. Although the two solutions are not strictly comparable (e.g., Ji and Pak consider Poisson's effects in the pile), the agreement of the results in Fig. 6 is evident.

### Strain Transmissibility

To examine the development of strain in the pile, it is useful to introduce the *strain transmissibility factor*

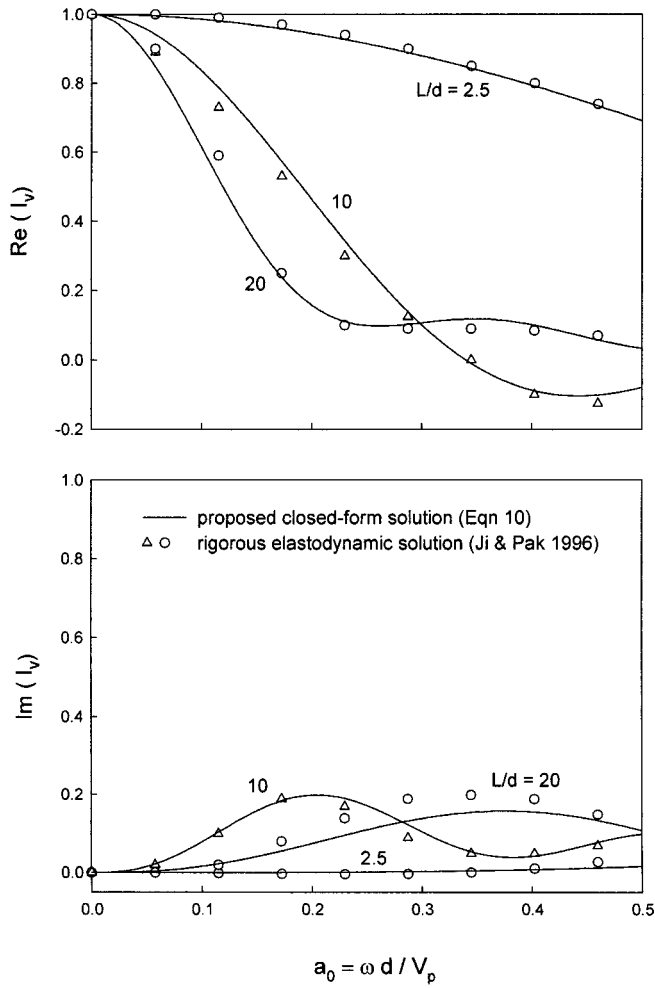
$$I_\varepsilon = \frac{(\varepsilon_p)_m}{(\varepsilon_s)_m} \quad (21)$$

which relates the maximum vertical normal strain in the pile  $(\varepsilon_p)_m$ , to the corresponding maximum normal soil strain  $(\varepsilon_s)_m$ . Differentiating Eq. (7) with respect to depth and introducing the peak soil strain  $(\varepsilon_s)_m = u_{\text{ffo}} g^*$ , the strain transmissibility is obtained as

$$I_\varepsilon = \frac{\Theta q^* \sin q^* L + \Omega \lambda (1 - \Theta) \cos q^* L}{\lambda (\Omega \cosh \lambda L + \sinh \lambda L)} \quad (22)$$

Eq. (22) is presented graphically in Fig. 7. It is seen that, contrary to the kinematic factor  $|I_v|$ ,  $|I_\varepsilon|$  may attain values higher than



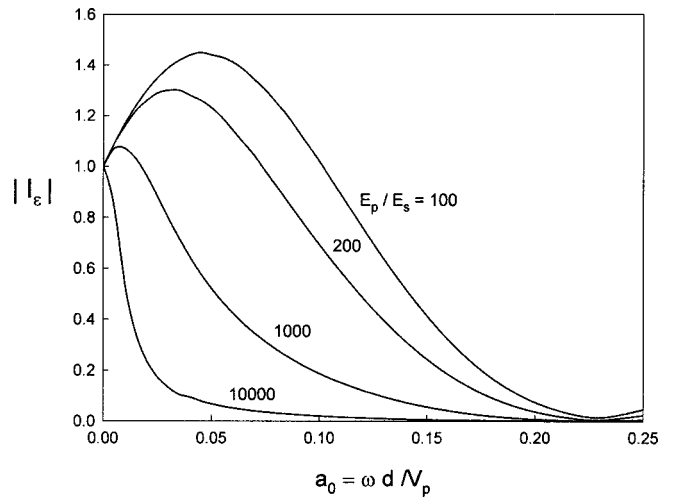


**Fig. 6.** Kinematic response coefficient  $I_v$ : Comparison of proposed solution with rigorous elastodynamic solution by Ji and Pak (1996);  $E_p/E_s = 960$ ,  $\rho_s/\rho_p = 0.25$ ,  $\nu_s = 0.25$ ,  $h/d = 0.025$ , and  $\beta = 0$

one, especially with soft piles. With increasing frequency, however,  $|I_\varepsilon|$  drops quickly and practically vanishes beyond  $a_0 = 0.20$ . Given that vertical soil strains during earthquakes are relatively small (i.e., typically less than  $10^{-4}$ ), it seems unlikely that this type of loading can inflict structural damage to the pile. Nevertheless, kinematic pile strains could possibly affect the safety of the pile when superimposed to other strains such as those due to dead loads, negative skin friction, inertial loads transmitted from the superstructure, etc.

## Group Effect

As mentioned in the Introduction, pile group effects on kinematic pile response are of minor importance. To understand this, it is instructive to view pile-to-pile interaction (PPI) as the result of the interplay of two distinct motions: (1) the vertical pile motion,  $u_p(z)$  and (2) the free-field soil motion at large distances from the pile,  $u_{ff}(z)$ . The difference between these two displacements,  $\delta u(z) = u_p(z) - u_{ff}(z)$ , can be looked upon as the origin of a scattered wave field which emanates from an oscillating pile and propagates through the soil until it reaches a neighboring pile. The scattered wave field imposes an additional (positive or nega-



**Fig. 7.** Strain transmissibility function  $I_\varepsilon$  for different pile-soil stiffness contrasts;  $L/d = 20$ ,  $H/L = 2$ ,  $\rho_s/\rho_p = 0.625$ ,  $\nu_s = 0.4$ , and  $\beta = 0.05$

tive depending on frequency) vertical displacement on the receiver pile which alters the dynamic response of the group.

In the light of the above concept, the minor importance of group action in the kinematic problem can be explained given the similarity between  $u_p(z)$  and  $u_{ff}(z)$ . Indeed, while for head-loaded piles  $u_{ff}$  is zero so  $\delta u(z) = u_p(z)$ , in the kinematic problem the deformed shape of the pile and the surrounding soil are very similar, so  $\delta u(z) = u_p(z) - u_{ff}(z) \ll u_p(z)$ , which ceases to generate a strong “scattered” wave field and, consequently, an appreciable group effect.

To quantify the phenomenon, it will be assumed, following Gazetas et al. (1992), that  $\delta u(z)$  can be approximated by

$$\delta u(z) \approx (\Theta - 1)u_{ff}(z) \quad (23)$$

which is based on the assumption that the particular solution of Eq. (2) is dominant. Considering that: (1)  $\delta u(z)$  attenuates with radial distance from a solitary pile approximately as a cylindrical wave and (2) the attenuated wave excites the base of the spring-dashpot bed of a receiver pile in the same manner as the free-field displacement  $u_{ff}(z)$  (Fig. 1), the additional vertical displacement of a receiver pile due to the scattered wave field is

$$\delta u_p(z) = \alpha I_v u_{ff}(z) \quad (24)$$

where  $\alpha = \alpha(r, \omega)$  is an *interaction factor* given by

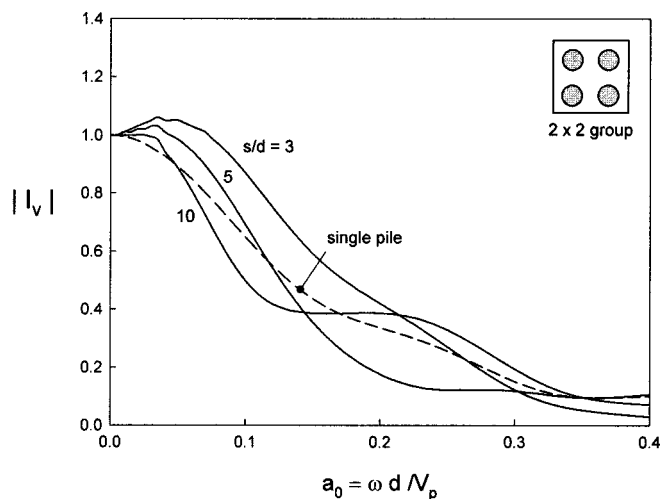
$$\alpha \approx \psi(r, \omega)(\Theta - 1) \quad (25)$$

in which  $\psi$  = approximate cylindrical wave function (Mylonakis and Gazetas 1998)

$$\psi(r, \omega) = \left(\frac{2r}{d}\right)^{-1/2} \exp\left[-(i + \beta_s)\left(\frac{r}{d} - \frac{1}{2}\right)\frac{\omega d}{V_s}\right] \quad (26)$$

Applying the superposition method of Poulos, the dynamic response of a pile group can be evaluated by considering the interaction between individual pile pairs. The method has been shown to exhibit good accuracy for dynamic loads by Kaynia and Kausel (1982). The method is well known and does not need to be explained here (see Dobry and Gazetas 1988); only results are presented below.

As an example, the kinematic response of a  $2 \times 2$  symmetric group,  $u_p^{(2 \times 2)}$ , is determined analytically as



**Fig. 8.** Kinematic response of  $2 \times 2$  pile group for various pile separations.  $E_p/E_s = 100$ ,  $L/d = 20$ ,  $H/L = 2$ ,  $\rho_s/\rho_p = 0.625$ ,  $\nu_s = 0.4$ , and  $\beta = 0.05$

$$u_p^{(2 \times 2)} = [1 + 2\alpha(s) + \alpha(\sqrt{2}s)]I_v u_{ff} \quad (27)$$

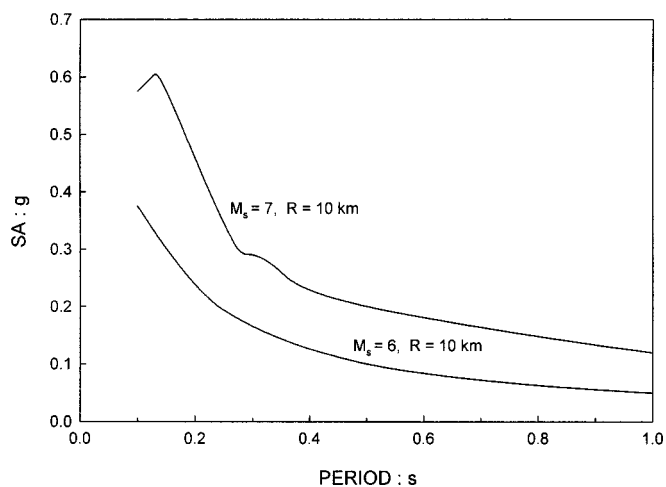
where  $s$  denotes the pile spacing.

Fig. 8 plots the ratio  $u_p^{(2 \times 2)}/u_{ff}$  for  $L/d = 20$ ,  $H/d = 40$ ,  $E_p/E_s = 100$ , and three pile separation distances,  $s/d = 3, 5$ , and  $10$ . The response of a single pile is also shown for comparison. Evidently PPI effects are not very important for a seismic excitation, contrary to head-loaded piles where PPI can be dominant. It should be mentioned, however, that this conclusion is strictly applicable only to homogeneous layers. As pointed out by Gazetas et al. (1992), heterogeneous deposits containing consecutive soil layers with sharply different stiffnesses are expected to behave differently and trigger stronger or weaker PPI effects.

## Some Practical Considerations

From the above developments, it is evident that kinematic interaction influences the pile head motion at dimensionless frequencies  $a_0$  higher than  $0.03$ , and may eliminate it almost entirely when  $a_0$  is about  $0.1$  to  $0.3$ . To investigate the practical significance of this effect, Fig. 9 presents average vertical recorded spectra on soft sites, processed by Ambraseys and Douglas (2000) for different magnitudes and source-to-site distances. It is seen that the range of predominant periods of the vertical spectra is between  $0.1$  and  $0.2$  s, which is much smaller than that of the horizontal motions. In addition, analytical evidence presented by Papazoglou and Elnashai (1996) and Elnashai and Papazoglou (1997) indicates that the fundamental natural period in the vertical direction  $T_v$ , of conventional reinforced concrete buildings can be less than  $0.2$  s even for structures with ten storeys, which coincides with the predominant periods of the vertical spectra. For tall steel buildings, Papaleontiou and Roesset (1993) found that  $T_v$  is approximately equal to  $1/10$ th of the fundamental natural period in lateral vibrations.

With reference to the compressional wave velocity in the soil, theory suggests that  $V_p$  in a perfectly saturated soil medium is just a little higher than the propagation velocity of P waves in water, i.e., about  $1500$ – $1800$  m/s (Richart, Woods, and Hall 1970). This result, however, should be used with caution. It is well known that a small deviation from perfect saturation may cause the propaga-



**Fig. 9.** Average recorded vertical acceleration spectra for soft soil sites (after Ambraseys and Douglas 2000)

tion velocity to drop substantially to about one third or less of the above value (Yang and Sato 2001). The effect of imperfect saturation was recently investigated by Yang and Sato (2000), to interpret the amplification patterns of vertical ground motion in Port Island, Kobe, during the severe earthquake of 1995. To match the recorded motions in the  $83$  m deep two-layer site, the above authors back-calculated compressional wave velocities of  $330$  and  $780$  m/s, in the two layers, respectively. The corresponding saturation ratios were estimated to range between  $98$  and  $99.6\%$ . The same authors also showed that using the ideal shear wave velocity of  $1,500$  m/s generates poor predictions of the recorded soil response.

Focusing in the dominant period range between  $0.1$  and  $0.2$  s ( $\omega = 62.8$  to  $31.4$  rad/s) and considering an average compressional wave velocity of about  $500$  m/s for an imperfectly saturated medium, the range of dimensional periods  $a_0$  for a  $d = 1$  m pile is approximately

$$a_0 \approx \frac{(31.4 \text{ to } 62.8) \times 1}{500} = 0.06 \text{ to } 0.13 \quad (28)$$

which is higher than the first transition frequency and may exceed the second one for long piles (Fig. 4). On the other hand, with small-diameter piles ( $d < 0.3$  m), the kinematic effect may be negligible. It is also worth mentioning that use of  $1,500$  m/s as compressional wave velocity would reduce the values in Eq. (28) by over  $60\%$ , leading to the erroneous conclusion that the vertical kinematic effect is insignificant.

## Conclusions

The most important findings of the study are: (1) The vertical motion of the pile head is always smaller than the corresponding free-field surface motion ( $|I_v| < 1$ ); (2) At dimensionless frequencies smaller than about  $0.03$  ( $a_{01}$ ), pile and the free-field soil move together. For frequencies higher than  $a_{01}$ , the motion of the pile head decreases quickly with increasing frequency. The decrease is faster with stiff piles; (3) Beyond a certain frequency ( $a_{02}$ ), the amplitude of pile motion fluctuates around an essentially constant value equal to approximately  $10$  to  $20\%$  of the free-field surface motion. For stiff piles ( $E_p/E_s > 200$ ), the transition frequency  $a_{02}$  is at least two times higher than the funda-

mental natural frequency of the layer in compression extension (Eq. 20). With soft piles ( $E_p/E_s < 200$ ),  $a_{02}$  may be twice as large; (4) Vertical normal strains in the pile can be larger than peak soil strains. Pile strain, however, decreases quickly with increasing frequency and practically vanish beyond about  $a_0 = 0.20$ ; (5) Group effects are of secondary importance for the kinematic problem; and (6) Considering the effect of imperfect soil saturation, vertical kinematic effects can be important for large diameter (e.g., bored) piles, but appear less significant for small-diameter piles.

## Acknowledgments

The writers would like to acknowledge the contribution of Mr. N. Kitrilakis and Dr. O. Michaelides in the early stages of this research. Thanks are also due to the anonymous reviewers for making useful suggestions for improving the quality of the manuscript. Finally, the help of Ms. Elia Voyagaki, Graduate student at CUNY, for preparing Fig. 6 and proofreading the final manuscript, is gratefully acknowledged.

## Appendix: Impedance Coefficients

The distributed soil springs and dashpots along the pile are given by (Gazetas et al. 1992)

$$k \approx 0.6E_s \left( 1 + \frac{1}{2} \sqrt{a_0 \frac{V_s}{V_p}} \right) \quad (29)$$

$$c \approx 1.2 \left( a_0 \frac{V_s}{V_p} \right)^{-1/4} \pi d \rho_s V_s + 2\beta k / \omega \quad (30)$$

where  $V_s$  = propagation velocity of shear waves in the soil. The dynamic impedance of the soil under the pile tip can be approximated as

$$K_b = \left| \frac{P}{u_p} \right|_{z=L} \approx \frac{E_s(1+2i\beta)d}{1-\nu_s^2} + i\omega\pi \left( \frac{d}{2} \right)^2 \rho_s V_{La} \quad (31)$$

where  $V_{La}$  = so-called "Lysmer's analog" wave velocity

$$V_{La} = \frac{3.4}{\pi(1-\nu_s)} V_s \quad (32)$$

Note that radiation damping (first term in the right side of Eq. (30) and second term in Eq. (31)) do not exist at frequencies lower than the fundamental natural frequency in compression extension of the soil layer.

## Notation

The following symbols are used in this paper:

- $A$  = pile cross-sectional area;
- $c$  = distributed vertical soil dashpot;
- $d$  = pile diameter;
- $E_p$  = pile Young's modulus;
- $E_s$  = soil Young's modulus;
- $g^*$  = complex  $P$ -wave number;
- $H$  = soil thickness;
- $h$  = wall thickness of hollow pile;
- $I_v$  = kinematic response factor;
- $I_e$  = strain transmissibility factor;

- $i = \sqrt{-1}$ ;
- $K_b$  = complex dynamic impedance at pile tip;
- $k$  = distributed vertical soil spring;
- $L$  = pile length;
- $m$  = pile mass per unit length;
- $P$  = axial pile force;
- $t$  = time;
- $u_{ff}$  = vertical soil displacement;
- $u_p$  = vertical pile displacement;
- $V_p$  = complex  $P$ -wave velocity in soil;
- $z$  = depth from soil surface;
- $\alpha$  = interaction factor;
- $\beta$  = soil material damping;
- $\varepsilon_p$  = vertical normal pile strain;
- $\varepsilon_s$  = vertical normal soil strain;
- $\Theta$  = dimensionless response parameter;
- $\lambda$  = Winkler parameter (complex pile wave number);
- $\nu_s$  = soil Poisson's ratio;
- $\rho_p$  = pile mass density;
- $\rho_s$  = soil mass density;
- $\Omega$  = dimensionless pile base impedance; and
- $\omega$  = cyclic vibrational frequency.

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